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CS 225

4.4 Structural Induction

7th edition 5.3: {12, 26c, 43, 44}

12) Let P(n) be f21 + f21 + ... + f2n = fnfn+1

Basis step: f21 = f1 \* f2 is true. f1  = 1, f2 = 1 1 \* 1 = 1 = f1 \* f2

Recursive step: Assume P(n) is true. P(n+1) has to be true to complete this step.

f21 + f21 + ... + f2n + fn+1 = (f21 + f22 + ... + f2n) + fn+1

= fnfn+1 + f2n+1

= fn+1(fn + fn+1)

= fn+1fn+2

26 c) basis step: 5 | (0 + 0) = 0

Recursive step: Let a+ b = 5k, k being an integer.

case 1: 5|(a+2) + (b+3) = a + b + 5 = 5k + 5 = 5(k+1), k+1 is also an integer

case 2: 5|(a+2) + (b + 2) = a + b + 4 = 5k + 5 = 5(k+1). This is the same as case 1

43 ) basis step: For a tree with only the root, it is true: n(T) = 1, h(T) = 0, 1 ≥ 2\*0 + 1

Recursive step: Need to show that n(T) ≥ 2h(T) + 1 for the full tree T. T is formed by T1 and T2 with addition, where T1 and T2 are smaller than T. The induction hypothesis shows that n(T1) ≥ 2h(T1) + 1 and n(T) = 1+n(T1) + n(T2) and h(T) = 1 + max(h(T1, h(T2))

n(T) = 1 + n(T1) + n(T2)

≥ 1 + 2h(T1) + 1 + 2h(T2) + 1

≥ 1 + 2max(h(T1), h(T2)) + 2

= 1 +2 (max(h(T1) , h(T2)) + 1)

= 1 + 2h(T)

44 ) Basis step: The single root r is the smallest tree. Since it has no internal vertices, it is a leaf. l(T) = 1 = 1 + i(T).

Recursive step: T is formed by T1 and T2.

l9T) = l(T1) + l(T2)

= i(T1) + 1 + i (T2) + 1

= i(T) + 1